

Lecture-0 (Motivation)

What is Discrete Mathematics

Discrete Mathematics is the branch of mathematics that deals with distinct, separate, and countable structures, rather than continuous ones. In contrast to calculus or real analysis, which study smooth change, discrete mathematics is concerned with objects that can be listed, counted, arranged, tabulated, or structured logically.

In short, Discrete Mathematics is a branch of Mathematics that deals with discrete objects (objects which are countable).

Examples of discrete objects:

1. People, Balls, Chairs, Tables.
2. The set of natural numbers \mathbb{N} , integers \mathbb{Z} , and rationals \mathbb{Q} .
3. Functions from $\{1, 2, \dots, n\}$ to $\{0, 1\}$.
4. Function $y = x^2$, where $(x, y) \in \mathbb{N} \times \mathbb{N}$.

The following objects are not discrete:

1. The real line \mathbb{R} , real plane \mathbb{R}^2 .
2. $y = x^2$ such that $(x, y) \in \mathbb{R}^2$.

Why do we study this subject separately.

Discrete mathematics forms the mathematical foundation of computer science. Every algorithm, data structure, cryptographic system, digital circuit, and database relies on discrete concepts. Key Reasons Why Discrete Mathematics Matters:

1. **Foundation of Computer Science:** Computers operate on binary data, logic, algorithms, and discrete structures, not continuous mathematics. Discrete math supplies the building blocks for:
 - Data structures (trees, heaps, graphs).
 - Algorithms (sorting, searching, optimization).

Thus, without discrete mathematics, computer science would not exist.

2. **Develops Logical & Analytical Thinking:** It teaches students to:
 - construct rigorous proofs.
 - reason step-by-step.
 - detect patterns, contradictions, and structure.
 - break complex problems into manageable parts.

Thus, these skills are essential for engineering, research, and problem-solving in general.

3. **Essential for Emerging Technologies:** Many cutting-edge technologies rely heavily on discrete mathematics such as:

Technology	Discrete Math Component
Cryptography & Cybersecurity	Number theory, modular arithmetic
AI & Machine Learning	Combinatorics, logic, graphs
Internet and Networking	Graph theory, shortest paths
Database & Information Systems	Sets, relations, logic
Blockchain	Hashing, prime arithmetic

4. **Helps in Efficient Decision Making:** Topics like combinatorics, optimization, graph theory, and probability help solve real-life problems:

- What is the fastest route for delivery trucks?
- How to schedule exams without clashes?
- How many ways can passwords be cracked?
- How to store and retrieve data efficiently?

Till class 12th, we have studied a part of discrete mathematics. Some topics are as follows.

1. Study of integers or natural numbers.
2. Arithmetic progression (AP), Geometric progression (GP), and Harmonic progression (HP).
3. Permutation and combination.

Discrete objects have some common characteristics and there is a set of common tools that are useful for handling discrete objects. So in discrete mathematics, we study this general set of tools. This give rise to number of subjects such as:

1. Finite set theory,
2. Combinatorics,
3. Finite group theory,
4. Discrete probability
5. Graph theory and many more.

In this course, we see some of the above topics in detail. Let us recall some puzzles that we likely encountered in our childhood. All these puzzles can be solved easily by using discrete mathematics.

1: Handshake Lemma

Puzzle: At a party, 7 people each shake hands with some of the others. Prove that the number of people who shook an odd number of hands is even.

2: Königsberg Bridge problem

Puzzle: Is it possible to cross each of the seven bridges exactly once and return to the start?

3: Missing Number (sum)

Puzzle: From $1 \cdots n$ one number is missing. Given the sum of the remaining, find the missing number.

4: Parity Handshake (3 odd hands)

Puzzle: With 5 people, can exactly 3 have shaken odd number of hands?